HONORS PROJECT: EARTHQUAKE INDUCED VIBRATIONS

MATH 251 – FALL 2020

For the study of earthquake induced vibrations on multistory buildings, let us assume that the free transverse oscillations satisfy a system of second order differential equations of the form

$$(*) \qquad m\mathbf{X}'' = kA\mathbf{X},$$

where A is an $n \times n$-matrix (for n floors), m is the mass of each floor, k denotes the stiffness of the columns supporting the floor, and $\mathbf{X}$ is the vector $(x_1(t), \ldots, x_n(t))$ describing the deformation of the columns.

On the i-th floor $(i \neq 1, n)$ the acting forces are

$$k(x_i - x_{i-1}) - k(x_{i+1} - x_i) = k(-x_{i-1} + 2x_i - x_{i+1})$$

giving rise to the differential equation

$$mx_i'' = k(x_{i-1} - 2x_i + x_{i+1}).$$

In the first and last floors the equations are $mx_1'' = k(-2x_1 + x_2)$ and $mx_n'' = k(x_n - x_{n-1})$, respectively. For a five-story building the matrix A from (*) becomes

$$A = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

1. Find the eigenvalues of A. To do this, find the characteristic polynomial $\varphi(\mu) = \det(A - \mu I)$ with help of the recursive formula from part 1 and use Newton's method to find its roots.

2. Use `sage` or `python` to find the eigenvectors of A.

3. Verify that if v is an eigenvector with eigenvalue λ, then for $\omega \in \mathbb{C}$ such that $\omega^2 = -\lambda \cdot k/m$ and for any constant α, the function
$$X(t) = \alpha \cos(\omega t) v$$
is a solution of (*).

4. Let $m = 1250$, $k = 10000$, and $\alpha = 0.075$. Take the eigenvalue of A with smallest absolute value, together with its eigenvector, and use them to get a solution vector $(x_1(t), \ldots, x_5(t))$ for (*).

5. Plot the parametric curves $x_k(t) + k \quad (k = 1, \ldots, 5)$ in one graph using different colors. What do these curves describe?